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## Effects of supersymmetric threshold corrections on high-scale flavour textures

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### Abstract

Integration of superpartners out of the spectrum induces potentially large contributions to Yukawa couplings. Supersymmetric threshold corrections therefore influence the CKM matrix prediction in a non-trivial way. We study the effects of threshold corrections on high-scale flavor structures specified at the gauge coupling unification scale in supersymmetry. We first consider high-scale Yukawa textures which qualify as phenomenologically viable at tree level, and find that they are disqualified after incorporating the threshold corrections. Next, we consider Yukawa couplings, such as those with five texture zeroes, which are incapable of explaining flavor-changing processes. Incorporating threshold corrections, however, makes them phenomenologically viable textures. Therefore, supersymmetric threshold corrections are found to have an observable impact on Yukawa couplings of quarks, and any confrontation of high-scale textures with experiments at the weak scale must take into account such corrections.

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### 1 Introduction and motivation

Supersymmetric theories with general soft breaking terms possess a number of flavor and CP violation sources [1]. In general, they are nested in the rigid and soft sectors of the theory, and bear no correlation or selection rules whatsoever. This is the case in supergravity and superstring scenarios where Kähler metric and superpotential couplings are all generic matrices in the space of fermion flavors and depend on the compactification scheme employed. This rather high degree of freedom in sources, textures and structures of the flavor mixings at GUT/string scale needs to be refined by confronting them with experimental data especially on rare processes. In general, testing high-scale flavor structures with experimental data involves three basic ingredients:

1. Specification of flavor textures in rigid and soft sectors at the messenger scale (which we take to be the MSSM gauge coupling unification scale  $Q = M_{GUT} \sim 10^{16}$  GeV).

2. Rescaling of lagrangian parameters to low-scale  $Q = M_{weak} \sim$  TeV via renormalization group flow. This stage is particularly important due to (i) largeness of the logs ( $\log M_{GUT}/M_{weak}$ ) involved, and (ii) modifications of flavor structures because of mixings with others.

3. Integration out of the superpartners at  $M_{weak}$  to achieve an effective theory which comprises the SM particle spectrum with possible imprints of supersymmetry in various couplings. For FCNC phenomenology this step is important as it induces flavor-nonuniversal couplings of gauge and Higgs bosons to fermions.

Any high-scale flavor structure specified in step 1 is classified to be phenomenologically viable if it agrees with experimental data after step 3. The first two steps have been widely discussed in literature by identifying flavor violation sources in general supergravity [2-5] and confronting them with experimental data on fermion masses and mixings as well as various observables in kaon and beauty systems [5-9].

So far analysis of the third step above has been restricted to TeV-scale supersymmetry where gauge [10] and Higgs [11,12] bosons have been found to develop flavor-changing couplings to fermions. In particular, emphasis has been put on the couplings of  $Z$  [10] and Higgs [13-18] to  $b\bar{s}$  since mixing between second and third generation fermions exhibits a theoretically clean and experimentally wide room for new physics. These analyses have led to conclusion that flavor violation sources in sfermion sector can have a big impact on Higgs phenomenology as well as various rare processes in kaon and beauty systems [11,12].

It is thus of prime phenomenological interest to know what impact the integration-out of sparticles can leave on high-scale flavor textures below  $M_{weak}$ . Stating more specifically, can integrating sparticles out of the spectrum render a given high-scale otherwise-viable texture inappropriate or generate CKM matrix from solely the soft sector or modify effects of Yukawa couplings on the soft sector? These are some of the questions which will be addressed in the present work.

In Sec. 2 below we briefly discuss the formalism for determining effects of supersymmetric threshold corrections. We mainly follow results of [11,12] therein. In Sec. 3, we first discuss in Sec. 3.1 sensitivities of the GUT-scale CKM-ruled, hierarchic and democratic Yukawa textures to supersymmetric threshold corrections when trilinear couplings are proportional to Yukawas. In Sec. 3.2 we investigate effects of flavor mixings in squark mass-squared matrices on textures analyzed in Sec. 3.1. In Sec. 3.3, we determine effects of threshold corrections on Yukawa textures which would not qualify physical at tree level. In Sec. 4 we conclude.

## 2 The formalism

The superpotential of the MSSM

$$\widehat{W} = \widehat{U}Y_u\widehat{Q}\widehat{H}_u + \widehat{D}Y_d\widehat{Q}\widehat{H}_d + \widehat{E}Y_e\widehat{L}\widehat{H}_d + \mu\widehat{H}_u\widehat{H}_d \quad (1)$$

encodes the rigid parameters  $\mu$  and Yukawa couplings  $Y_{u,d,e}$  (of up quarks, down quarks and of leptons) each being a  $3 \times 3$  non-hermitian matrix in the space of fermion flavors.

The breakdown of supersymmetry is parameterized by a set of soft (*i.e.* operators of dimension  $\leq 3$ ) terms [1]

$$\begin{aligned} \mathcal{L}_{soft} = & m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + \tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} + \tilde{U} \mathbf{m}_U^2 \tilde{U}^\dagger + \tilde{D} \mathbf{m}_D^2 \tilde{D}^\dagger + \tilde{L}^\dagger \mathbf{m}_L^2 \tilde{L} + \tilde{E} \mathbf{m}_E^2 \tilde{E}^\dagger \\ & + \left[ \tilde{U} Y_u^A \tilde{Q} H_u + \tilde{D} Y_d^A \tilde{Q} H_d + \tilde{E} Y_e^A \tilde{L} H_d + \mu B H_u H_d + \frac{1}{2} \sum_{\alpha} M_{\alpha} \lambda_{\alpha} \lambda_{\alpha} + \text{h.c.} \right] \end{aligned} \quad (2)$$

where trilinear couplings  $Y_{u,d,e}^A$  like Yukawas themselves are non-hermitian flavor matrices whereas the sfermion mass-squareds  $\mathbf{m}_{Q,\dots,E}^2$  are all hermitian. In general, all of the parameters in the second line and off-diagonal entries of the sfermion mass-squared matrices are endowed with CP-odd phases; they serve as sources of CP violation beyond the SM. The Yukawa matrices, trilinear couplings and sfermion mass-squareds facilitate flavor violation in processes mediated by sparticle loops. The MSSM possesses 21 mass parameters, 36 mixing angles and 40 CP-odd phases in addition to ones in the SM [19]. Consequently, there is a 97-dimensional parameter space to be scanned in confronting theory with experiments at  $M_{weak}$ . In supergravity or string models the parameters of (1) and (2) are determined by compactification mechanism and structure of the internal manifold [2-4].

The parameters of (1) and (2) are scale- dependent. They are rescaled to  $Q = M_{weak}$  via the MSSM RGEs [20-24] with boundary conditions specified at  $Q = M_{GUT}$ . The RG running of model parameters is crucial. In fact, various phenomena central to supersymmetry phenomenology *e.g.* gauge coupling unification, radiative electroweak breaking, induction of flavor structures even for flavor-blind soft terms are pure renormalization effects. The Yukawa couplings,  $\mu$  parameter and gauge couplings form a coupled closed set of observables [25] in that their scale dependencies are not affected by soft-breaking sector unless some sparticles are decoupled before reaching  $M_{weak}$ . On the other hand, running of the soft masses depend explicitly on rigid parameters of the theory, and they develop, among other things, novel flavor structures thanks to the Yukawa matrices. For instance, evolution of the soft mass-squared of left-handed squarks

$$\begin{aligned}
 \frac{d\mathbf{m}_Q^2}{dt} &= \mathbf{m}_Q^2 \left( \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d \right) + \left( \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d \right) \mathbf{m}_Q^2 \\
 &+ 2 \left( \mathbf{Y}_u^\dagger \mathbf{m}_U^2 \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{m}_D^2 \mathbf{Y}_d + \mathbf{Y}_u^{A\dagger} \mathbf{Y}_u^A + \mathbf{Y}_d^{A\dagger} \mathbf{Y}_d^A \right) \\
 &+ 2 \left( m_{H_u}^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + m_{H_d}^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \right) \\
 &- 2 \left( \frac{16}{3} g_3^2 |M_3|^2 + 3g_2^2 |M_2|^2 + \frac{1}{15} g_1^2 |M_1|^2 \right) \mathbf{1},
 \end{aligned} \tag{3}$$

with  $t = (4\pi)^{-2} \log Q/M_{GUT}$ , shows explicitly how flavor structure of a given parameter, say  $\mathbf{m}_Q^2$ , at a given scale  $Q$  senses those of the remaining parameters. Indeed, flavor mixings exhibited by  $\mathbf{m}_Q^2$  at  $Q = M_{weak}$  can stem from  $\mathbf{m}_{Q,U,D}^2$  or  $\mathbf{Y}_{u,d}$  or  $\mathbf{Y}_{u,d}^A$  or all of them. Therefore, a given pattern of flavor mixings in, for instance, kaon system can be sourced by various flavor matrices in rigid as well as soft sectors of the theory.

The flavor structures at  $M_{weak}$  arising from solutions of RGEs are further rehabilitated by taking into account the decoupling of sparticles at the supersymmetric threshold. Indeed, once part of the sparticles are integrated out of the spectrum the effective theory below  $M_{weak}$  can exhibit sizeable non-standard effects in certain scattering channels of the SM particles [10-18]. Taking the effective theory below  $M_{weak}$  to be two-Higgs-doublet model (2HDM) one finds

$$\begin{aligned}
 \mathbf{Y}_d^{eff} &= \mathbf{Y}_d(M_{weak}) - \gamma^d + \tan \beta \Gamma^d \\
 \mathbf{Y}_u^{eff} &= \mathbf{Y}_u(M_{weak}) - \gamma^u + \cot \beta \Gamma^u
 \end{aligned} \tag{4}$$

where  $\mathbf{Y}_{d,u}(M_{weak})$  are solutions of the corresponding RGEs evaluated at  $Q = M_{weak}$ , and  $\gamma^{d,u}$  and  $\Gamma^{d,u}$  are flavor matrices arising from squark-gluino and squark-Higgsino loops. Their explicit expressions can be found in [11,12].

The physical quark fields are obtained by rotating the original gauge eigenstate fields via the unitary matrices  $V_{R,L}^{u,d}$  that diagonalize  $\mathbf{Y}_{u,d}^{eff}$ :

$$\left( V_R^d \right)^\dagger \mathbf{Y}_d^{eff} V_L^d = \overline{\mathbf{Y}}_d, \quad \left( V_R^u \right)^\dagger \mathbf{Y}_u^{eff} V_L^u = \overline{\mathbf{Y}}_u \tag{5}$$

where  $\overline{\mathbf{Y}}_d = \text{diag.}(\overline{h}_d, \overline{h}_s, \overline{h}_b)$  and  $\overline{\mathbf{Y}}_u = \text{diag.}(\overline{h}_u, \overline{h}_c, \overline{h}_t)$  are physical Yukawa matrices whose entries are directly related to running quark masses at  $Q = M_{weak}$ :

$$\overline{h_u} = \frac{g_2(M_{weak})m_u(M_{weak})}{\sqrt{2}M_W \sin \beta}, \quad \overline{h_d} = \frac{g_2(M_{weak})m_d(M_{weak})}{\sqrt{2}M_W \cos \beta} \quad (6)$$

with similar expressions for other generations.

In general, whatever flavor textures are adopted at  $M_{GUT}$ , the resulting CKM matrix,  $V_{CKM}^{corr} \equiv (V_L^u)^\dagger V_L^d$ , must agree with the existing experimental bounds [26]. Clearly, in the limit of vanishing threshold corrections  $\Gamma^{u,d}$  and  $\gamma^{u,d}$ , physical CKM matrix  $V_{CKM}^{corr}$  reduces to  $V_{CKM}^{tree}$  computed by diagonalizing  $Y_{u,d}(M_{weak})$ . Reiterating, it is with comparison of the predicted CKM matrix,  $V_{CKM}^{corr}$ , with experiment that one can tell if a high-scale texture, classified to be viable at tree-level by considering  $V_{CKM}^{tree}$  only, is spoiled by the supersymmetric threshold corrections. The experimental bounds on the absolute magnitudes of the CKM entries (at 90% CL) read collectively as:

$$|V_{CKM}^{exp}| = \left( \begin{array}{cc|cc|cc} \boxed{0.9739} \boxed{0.9751} & \boxed{0.2210} \boxed{0.2270} & \boxed{0.0029} \boxed{0.0045} \\ \boxed{0.2210} \boxed{0.2270} & \boxed{0.9730} \boxed{0.9744} & \boxed{0.0390} \boxed{0.0440} \\ \boxed{0.0048} \boxed{0.0140} & \boxed{0.0370} \boxed{0.0430} & \boxed{0.9990} \boxed{0.9992} \end{array} \right) \quad (7)$$

where left (right) window of  $\boxed{\quad\quad}$  in each entry refers to lower (upper) experimental bound on the associated CKM element. Clearly, the largest uncertainty occurs in  $|V_{td}|$ . These matrix elements are measured at  $Q = M_Z$ , and for a comparison with predictions of the effective theory below  $Q = M_{weak}$  they have to be scaled from  $M_Z$  up to  $M_{weak}$ . This can be done without having a detailed knowledge of the particle spectrum of the effective 2HDM at  $M_{weak}$  (as emphasized above, the effective theory may consist of some light superpartners in which case beta functions of certain couplings get modified as exemplified by analyses of  $b \rightarrow s\gamma$  decay in effective supersymmetry [27,28]) since RG running of the CKM elements is such that  $V_{CKM}(1, 1)$ ,  $V_{CKM}(1, 2)$ ,  $V_{CKM}(2, 1)$ ,  $V_{CKM}(2, 2)$  and  $V_{CKM}(3, 3)$  do not evolve with energy scale, to an excellent approximation [29-31]. Therefore, it is rather safe to confront the CKM matrix predicted by the effective theory at  $M_{weak}$  with the experimental results (7) entry by entry excluding, however,  $V_{CKM}(1, 3)$ ,  $V_{CKM}(3, 1)$ ,  $V_{CKM}(2, 3)$  and  $V_{CKM}(3, 2)$  for which renormalization effects can be sizeable.

In the next section, we will compute supersymmetric threshold corrections to Yukawa couplings of quarks for certain prototype flavor textures defined at  $Q = M_{GUT}$ . In particular, we will evaluate radiatively corrected CKM matrix as well as couplings of the Higgs bosons to quarks to

determine the impact of the decoupling of squarks out of the spectrum at  $M_{weak}$  on scattering processes at energies accessible to present and future colliders.

### 3 High-scale textures and threshold corrections

First of all, for standardization and easy comparison with literature (*e.g.* with the computer codes ISAJET [32] and SOFTSUSY [33]) we take SPS1a' conventions for supersymmetric parameters [34]

$$\tan \beta = 10, m_0 = 70 \text{ GeV}, A_0 = -300 \text{ GeV}, m_{1/2} = 250 \text{ GeV} \quad (8)$$

and completely neglect supersymmetric CP-violating phases, as mentioned before.

Instead of scanning a 97-dimensional parameter space for specifying what high-scale parameter ranges are useful for what low-energy observables, which is actually what has to be done, we simplify the analysis by focussing on certain prototype textures at high scale. In general, for any flavor matrix in any sector of the theory there exist, boldly speaking, three extremes: (*i*) completely diagonal, (*ii*) hierarchical, and (*iii*) democratic textures. There are, of course, a continuous infinity of textures among these extremes; however, for definiteness and clarity in our analysis we will focus on these three structures.

#### 3.1 Flavor violation from Yukawas and trilinear couplings

In this subsection we investigate effects of supersymmetric threshold corrections on high-scale textures in which Yukawa couplings exhibit non-trivial flavor mixings and so do the trilinear couplings since we take

$$\mathbf{Y}_{u,d,e}^A = A_0 \mathbf{Y}_{u,d,e} \quad (9)$$

at the GUT scale. The soft mass-squareds, on the other hand, are taken entirely flavor conserving *i.e.* they are strictly diagonal and universal at the GUT scale. It is with direct proportionality of trilinear couplings with Yukawas and certain ansätze for Yukawa textures that, we will study below sensitivities of certain high-scale Yukawa structures to supersymmetric threshold corrections at the TeV scale.

##### 3.1.1 CKM-ruled texture

We take Yukawa couplings of up and down quarks to be

$$\begin{aligned}
 \mathbf{Y}_u &= \text{diag}(3.5 \cdot 10^{-6}, 1.3 \cdot 10^{-3}, 0.4566) \\
 \mathbf{Y}_d &= \begin{pmatrix} 6.2368 \cdot 10^{-5} & -1.4272 \cdot 10^{-5} & 5.9315 \cdot 10^{-7} e^{0.3146i} \\ 2.4640 \cdot 10^{-4} & 1.07074 \cdot 10^{-3} & -4.0458 \cdot 10^{-5} \\ 1.6495 \cdot 10^{-4} e^{1.047i} & 1.81465 \cdot 10^{-3} & 4.8476 \cdot 10^{-2} \end{pmatrix} \quad (10)
 \end{aligned}$$

with no flavor violation in the lepton sector:  $\mathbf{Y}_e = \text{diag.}(1.9 \cdot 10^{-5}, 4 \cdot 10^{-3}, 0.071)$ . The flavor violation effects are entirely encoded in  $\mathbf{Y}_d$  which exhibits a CKM-ruled hierarchy in similarity to Yukawa textures analyzed in [5] *i.e.* this choice of boundary values of the Yukawas leads to correct CKM matrix [26] at  $M_{weak}$  upon integration of the RGEs.

At the weak scale the Yukawa matrices, trilinear couplings and squark soft mass-squareds serve as sources of flavor violation. The trilinear couplings, under two-loop RG running [20-24] with boundary conditions (9), attain the flavor structures

$$\begin{aligned}
 \mathbf{Y}_u^A &= \begin{pmatrix} -7.2 \cdot 10^{-3} & 0 & 0 \\ 1.70 \cdot 10^{-6} e^{0.5641i} & -2.67 & 2.9 \cdot 10^{-4} \\ 6.24 e^{1.047i} \cdot 10^{-3} & 6.8 \cdot 10^{-2} & -532.7 \end{pmatrix} \\
 \mathbf{Y}_d^A &= \begin{pmatrix} -0.204 & -0.191 & -0.138 e^{-1.039i} \\ -0.567 & -3.495 & -1.436 \\ -0.384 e^{1.046i} & -4.19 & -134.24 \end{pmatrix} \quad (11)
 \end{aligned}$$

both measured in GeV at  $M_{weak} = 1 \text{ TeV}$ . Clearly,  $\mathbf{Y}_u^A$  is essentially diagonal whereas (2, 3), (3, 2) and (2, 2) entries of  $\mathbf{Y}_d^A$  are of the same size.

Though they start with completely diagonal and universal boundary values, the squark soft squared masses develop flavor-changing entries at  $M_{weak} = 1 \text{ TeV}$ :

$$\begin{aligned}
 \mathbf{m}_Q^2 &= (533.67 \text{ GeV})^2 \begin{pmatrix} 1.07 & 0.0 & 0.0 \\ 0.0 & 1.07 & -2.2 \cdot 10^{-4} \\ 0.0 & -2.2 \cdot 10^{-4} & 0.86 \end{pmatrix} \\
 \mathbf{m}_D^2 &= (530.76 \text{ GeV})^2 \begin{pmatrix} 1.01 & 0.0 & 0.0 \\ 0.0 & 1.01 & -1.5 \cdot 10^{-4} \\ 0.0 & -1.5 \cdot 10^{-5} & 0.99 \end{pmatrix} \quad (12)
 \end{aligned}$$

with  $\mathbf{m}_{\tilde{U}}^2 = (497.11 \text{ GeV})^2 \text{diag.} (1.15, 1.15, 0.69)$ . The numerical values of the parameters above exhibit good agreement with well-known codes like ISAJET [32] and SOFTSUSY [33].

The presence of flavor violation in the soft sector of the low-energy theory gives rise to non-trivial corrections to Yukawa couplings and in turn to the CKM matrix. Indeed, use of (11) and (12) in [11,12] introduces certain corrections to the tree-level Yukawa matrices  $\mathbf{Y}_{u,d}(M_{weak})$  to generate  $\mathbf{Y}_{u,d}^{eff}$  in (4). In fact,  $V_{CKM}^{tree}$  (obtained from  $\mathbf{Y}_{u,d}(M_{weak})$ ) and  $V_{CKM}^{corr}$  (obtained from  $\mathbf{Y}_{u,d}^{eff}$ ) compare to exhibit spectacular differences:

$$\left[ \begin{array}{c|c} V_{CKM}^{tree} & V_{CKM}^{corr} \end{array} \right] = \left( \begin{array}{cc|cc|cc} \hline 0.9746 & 0.9795 & 0.2241 & 0.2015 & 0.0037 & 0.0034 & \hline \hline 0.2240 & 0.2014 & 0.9737 & 0.9788 & 0.0406 & 0.0375 & \hline \hline 0.0079 & 0.0066 & 0.0400 & 0.0371 & 0.99917 & 0.9993 & \hline \end{array} \right) \quad (13)$$

where left (right) window of  $\left[ \begin{array}{c|c} \dots & \dots \end{array} \right]$  in  $(i, j)$ -th entry refers to  $|V_{CKM}^{tree}(i, j)|$  ( $|V_{CKM}^{corr}(i, j)|$ ). Clearly,  $|V_{CKM}^{tree}|$  agrees very well with  $|V_{CKM}^{exp}|$  in (7) entry by entry. This qualifies (10) to be the correct high-scale texture given experimental FCNC bounds at  $Q = M_Z$ . However, radiative corrections induced by decoupling of squarks, gluinos and Higgsinos at the supersymmetric threshold  $M_{weak} = 1 \text{ TeV}$  is seen to leave a rather strong impact on the CKM entries. Consider for instance  $(1, 1)$  entries of  $V_{CKM}^{exp}$ ,  $V_{CKM}^{tree}$  and  $V_{CKM}^{corr}$ . Present experiments provide a  $1.64\sigma$  significance to  $|V_{CKM}^{exp}(1, 1)|$  around a mean value of 0.745 as is seen from (7). The tree-level prediction,  $|V_{CKM}^{tree}(1, 1)|$ , takes the value of 0.9746 which is rather close to the center of the experimental interval. However, once supersymmetric threshold corrections are included this tree-level prediction gets modified to  $|V_{CKM}^{corr}(1, 1)| = 0.9795$ . This value is obviously far beyond the existing experimental limits as it is a  $13.39\sigma$  effect. Similarly,  $|V_{CKM}^{corr}(1, 2)|$ ,  $|V_{CKM}^{corr}(2, 1)|$ ,  $|V_{CKM}^{corr}(2, 2)|$  and  $|V_{CKM}^{corr}(3, 3)|$  are, respectively,  $12.36\sigma$ ,  $12.36\sigma$ ,  $11.95\sigma$  and  $2.30\sigma$  effects. Obviously, deviation of  $|V_{CKM}^{corr}(i, j)|$  from  $|V_{CKM}^{tree}(i, j)|$  (comparison with experiments at  $Q = M_Z$  is meaningful especially for  $(i, j) = (1, 1)$ ,  $(1, 2)$ ,  $(2, 1)$ ,  $(3, 3)$  entries whose scale dependencies are known to be rather mild [29-31]), when the latter falls well inside the experimentally allowed range, obviously violates existing experimental bounds in (7) by several standard deviations. Consequently, supersymmetric threshold corrections entirely disqualify the high-scale texture (10) being the correct texture to reproduce the FCNC measurements at the weak scale. This case study, based on numerical values for Yukawa entries in (10), manifestly shows the impact of supersymmetric threshold corrections on high-scale textures which qualify viable at tree level.

The physical quark fields, which arise after the unitary rotations (5), acquire the masses

$$\overline{\mathbf{M}}_{\mathbf{u}}(M_{weak}) = \text{diag.}(\approx 0, 0.545, 149.45), \quad \overline{\mathbf{M}}_{\mathbf{d}}(M_{weak}) = \text{diag.}(3.35 \cdot 10^{-3}, 5.76 \cdot 10^{-2}, 2.33) \quad (14)$$

all measured in GeV. In this physical basis for quark fields,  $V_{CKM}^{corr}$  governs the strength of charged current vertices for each pair of up and down quarks. These mass predictions are to be evolved down to  $Q = M_Z$  to make comparisons with experimental results. This evolution depends on the effective theory below  $M_{weak}$ . Speaking conversely, the high-scale texture (10) as to be folded in such a way that resulting mass and mixing patterns for quarks agree with experiments below the sparticle threshold  $M_{weak}$ .

### 3.1.2 Hierarchical texture

The Yukawa couplings are taken to have the structure (as can be motivated from [35-38])

$$\mathbf{Y}_{\mathbf{u}} = \begin{pmatrix} 2.6463 \cdot 10^{-4} & 5.8163 \cdot 10^{-4}i & -1.0049 \cdot 10^{-2} \\ -5.8163 \cdot 10^{-4}i & 2.2587 \cdot 10^{-3} & 1.0049 \cdot 10^{-5}i \\ -4.8233 \cdot 10^{-3} & -9.0437 \cdot 10^{-6}i & 0.495 \end{pmatrix} \quad (15)$$

$$\mathbf{Y}_{\mathbf{d}} = \begin{pmatrix} 3.9808 \cdot 10^{-4} & 8.1167 \cdot 10^{-4} e^{0.734i} & -1.1431 \cdot 10^{-3} \\ 8.1167 \cdot 10^{-4} e^{-0.734i} & 2.7997 \cdot 10^{-3} & 2.04844 \cdot 10^{-3}i \\ -1.1431 \cdot 10^{-3} & -1.6461 \cdot 10^{-3}i & 4.97 \cdot 10^{-2} \end{pmatrix}$$

with no flavor violation in the lepton sector:  $\mathbf{Y}_e = \text{diag.}(1.9 \cdot 10^{-5}, 0.004, 0.071)$ . Here both  $\mathbf{Y}_{\mathbf{u}}$  and  $\mathbf{Y}_{\mathbf{d}}$  exhibit a hierarchically organized pattern of entries. In a sense, the hierarchic nature of  $\mathbf{Y}_{\mathbf{d}}$  in (10) is now extended to  $\mathbf{Y}_{\mathbf{u}}$  so as to form a complete hierarchic pattern for quark Yukawas at the GUT scale.

At the weak scale, the Yukawa matrices above, trilinear couplings, and squark soft mass-squareds serve as sources of flavor violation. The trilinear couplings, under two-loop RG running [20-24] with boundary conditions (9), obtain the flavor structures

$$\begin{aligned}
 \mathbf{Y}_u^A &= \begin{pmatrix} -0.4315 & -1.1442i & 10.637 \\ 1.1466i & -4.4531 & -4.8631 \cdot 10^{-3}i \\ 5.0657 & -0.1046i & -524.07 \end{pmatrix} \\
 \mathbf{Y}_d^A &= \begin{pmatrix} -1.2934 & -2.6494 e^{0.734i} & 3.1221 \\ 2.6428 e^{-0.731i} & -9.1395 & 5.2606i \\ 3.4532 & -5.6827i & -135.861 \end{pmatrix}
 \end{aligned} \tag{16}$$

both measured in GeV at  $M_{weak} = 1$  TeV. Clearly, in contrast to (11), now both  $\mathbf{Y}_u^A$  and  $\mathbf{Y}_d^A$  develop sizeable off-diagonal entries, as expected from (15).

Though they start with completely diagonal and universal boundary values, the squark soft squared masses develop flavor-changing entries at  $M_{weak} = 1$  TeV:

$$\begin{aligned}
 \mathbf{m}_Q^2 &= (533.69 \text{ GeV})^2 \begin{pmatrix} 1.07 & 1.9 \cdot 10^{-5} e^{1.144i} & 2.14 \cdot 10^{-3} \\ 1.9 \cdot 10^{-5} e^{-1.144i} & 1.07 & 3.17 \cdot 10^{-4}i \\ 2.14 \cdot 10^{-3} & -3.17 \cdot 10^{-4}i & 0.86 \end{pmatrix} \\
 \mathbf{m}_U^2 &= (496.76 \text{ GeV})^2 \begin{pmatrix} 1.16 & -6.66 \cdot 10^{-6}i & 9.6 \cdot 10^{-3} \\ 6.66 \cdot 10^{-6}i & 1.16 & -1.4 \cdot 10^{-5}i \\ 9.6 \cdot 10^{-3} & 1.4 \cdot 10^{-5}i & 0.685 \end{pmatrix} \\
 \mathbf{m}_D^2 &= (531.07 \text{ GeV})^2 \begin{pmatrix} 1.01 & 3.3 \cdot 10^{-5} e^{1.06i} & 3.75 \cdot 10^{-4} \\ 3.3 \cdot 10^{-5} e^{-1.06i} & 1.01 & -6.62 \cdot 10^{-4}i \\ 3.75 \cdot 10^{-4} & 6.62 \cdot 10^{-4}i & 0.99 \end{pmatrix}
 \end{aligned} \tag{17}$$

whose average values show good agreement with (12) but certain off-diagonal entries exhibit significant enhancements when the corresponding entries of Yukawas and trilinear couplings are sizeable.

The flavor-violating entries of Yukawas, trilinear couplings and soft mass-squareds collectively generate radiative contributions  $\gamma^{\mu,d}$ ,  $\Gamma^{u,d}$  to the Yukawa couplings below  $M_{weak}$  [11,12]. In fact,  $V_{CKM}^{tree}$  (obtained from  $\mathbf{Y}_{u,d}(M_{weak})$ ) and (obtained from  $\mathbf{Y}_{u,d}^{eff}$  confront as follows:

$$\left\| V_{CKM}^{tree} \right\| \left\| V_{CKM}^{corr} \right\| = \begin{pmatrix} \boxed{0.9745} \boxed{0.9773} & \boxed{0.2243} \boxed{0.2118} & \boxed{0.0049} \boxed{0.0034} \\ \boxed{0.2240} \boxed{0.2116} & \boxed{0.9737} \boxed{0.9766} & \boxed{0.0417} \boxed{0.0379} \\ \boxed{0.0109} \boxed{0.0091} & \boxed{0.0405} \boxed{0.0370} & \boxed{0.99912} \boxed{0.99927} \end{pmatrix} \tag{18}$$

where left (right) window of  $\square\square$  in  $(i, j)$ -th entry refers to  $|V_{CKM}^{tree}(i, j)|$  ( $|V_{CKM}^{corr}(i, j)|$ ). Clearly,  $|V_{CKM}^{tree}|$  falls well inside the  $1.64\sigma$  experimental interval in (7) entry by entry. In this sense, Yukawa matrices in (20) qualify to be the correct high-scale textures given present experimental determination of  $V_{CKM}$  at  $Q = M_Z$ . However, this agreement between experiment and theory gets spoiled strongly by the inclusion of supersymmetric threshold corrections. Indeed, as is shown comparatively by (23),  $V_{CKM}^{corr}$  violates the bounds in (7) significantly. More precisely,  $|V_{CKM}^{corr}(1, 1)|$ ,  $|V_{CKM}^{corr}(1, 2)|$ ,  $|V_{CKM}^{corr}(2, 1)|$ ,  $|V_{CKM}^{corr}(2, 2)|$ ,  $|V_{CKM}^{corr}(3, 3)|$  turn out to have  $7.65\sigma$ ,  $6.83\sigma$ ,  $6.77\sigma$ ,  $6.79\sigma$ ,  $3.28\sigma$  significance levels, respectively. These significance levels are far beyond the existing experimental  $1.64\sigma$  intervals depicted in (7). As a result, supersymmetric threshold corrections are found to entirely disqualify the high-scale texture (15) to be the correct texture to reproduce the FCNC measurements at the weak scale. This case study therefore shows the impact of supersymmetric threshold corrections on high-scale textures which qualify viable at tree level.

The physical quark fields, which arise after the unitary rotations (5), acquire the masses

$$\overline{\mathbf{M}}_{\mathbf{u}}(M_{weak}) = \text{diag.}(0.0065, 0.98, 153.82), \quad \overline{\mathbf{M}}_{\mathbf{d}}(M_{weak}) = \text{diag.}(0.0071, 0.155, 2.37) \tag{19}$$

all measured in GeV. In this physical basis for quark fields,  $V_{CKM}^{corr}$  is responsible for charged current interactions in the effective theory below  $M_{weak}$ . The morale of the analysis above is that, the high-scale flavor structures (15) are to be modified in such a way that  $V_{CKM}^{corr}$  agrees with  $V_{CKM}^{exp}$  with sufficient precision. Aftermath, the question is to predict quark masses appropriately at  $Q = M_{weak}$  so that, depending on the particle spectrum of the effective theory beneath, existing experimental values of quark masses at  $Q = M_Z$  are reproduced correctly.

### 3.1.3 Democratic texture

In this subsection, we take Yukawa couplings to be (as can be motivated from relevant works [39-41])

$$\mathbf{Y}_{\mathbf{u}} = \begin{pmatrix} 0.1475 & 0.1443 & 0.1458 \\ 0.1443 & 0.1475 & 0.1458 \\ 0.1456 & 0.1458 & 0.1456 \end{pmatrix}$$

$$\mathbf{Y}_{\mathbf{d}} = \begin{pmatrix} 0.01583 & 0.01452(1 - 10^{-2}i) & 0.01553(1 - 10^{-2}i) \\ 0.01452(1 + 10^{-2}i) & 0.01944 & 0.01617(1 + 2 \cdot 10^{-2}i) \\ 0.01551(1 + 10^{-2}i) & 0.01617(1 - 2 \cdot 10^{-2}i) & 0.01604 \end{pmatrix} \tag{20}$$

with no flavor violation in the lepton sector:  $Y_e = \text{diag.} (1.9 \cdot 10^{-5}, 4 \cdot 10^{-3}, 0.071)$ . Here both  $Y_u$  and  $Y_d$  exhibit an approximate democratic structure so that  $Y_{u,d}(M_{weak})$  generate correctly masses and mixings of the quarks at the weak scale. Clearly, in the exact democratic limit two of the quarks from each sector remain massless, and therefore, a realistic flavor structure is likely to come from small perturbations of the exact democratic texture [39-41]. Another important feature of exact democratic texture is that all higher powers of Yukawas reduce to Yukawas themselves up to a multiplicative factor, and this gives rise to linearization of and in turn direct solution of Yukawa RGEs in the form of an RG rescaling of the GUT scale texture [25]. These properties remain approximately valid for perturbed democratic textures like (20).

At the weak scale, the Yukawa matrices above, trilinear couplings, and squark soft mass-squareds serve as sources of flavor violation. The trilinear couplings, under two-loop RG running [20-24] with boundary conditions (9), obtain the flavor structures

$$\begin{aligned}
 Y_u^A &= - \begin{pmatrix} 182.44 & 175.57 & 178.81 \\ 175.69 & 182.32 & 178.81 \\ 178.62 & 178.81 & 178.67 \end{pmatrix} \\
 Y_d^A &= - \begin{pmatrix} 44.41 & 40.07 e^{-0.0117i} & 43.39 e^{0.0115i} \\ 39.44 e^{0.0101i} & 55.46 e^{-0.0013i} & 44.82 e^{0.0218i} \\ 43.09 e^{-0.099i} & 45.17 e^{-0.0216i} & 44.79 e^{0.0016i} \end{pmatrix}
 \end{aligned} \tag{21}$$

both measured in GeV at  $M_{weak} = 1 \text{ TeV}$ . Though not shown explicitly, each entry of  $Y_u^A$  is complex with a phase around  $10^{-7} - 10^{-6}$  in size.

Though they start with completely diagonal and universal boundary values, the squark soft squared masses develop flavor-changing entries at  $M_{weak} = 1 \text{ TeV}$ :

$$\begin{aligned}
 m_Q^2 &= (533.67 \text{ GeV})^2 \begin{pmatrix} 1.0 & 0.0672 & 0.0670 \\ 0.0672 & 1.0 & 0.0673 \\ 0.0670 & 0.0673 & 1.0 \end{pmatrix} \\
 m_U^2 &= (497.38 \text{ GeV})^2 \begin{pmatrix} 1.0 & 0.1526 & 0.1524 \\ 0.1526 & 1.0 & 0.1524 \\ 0.1524 & 0.1524 & 1.0 \end{pmatrix} \\
 m_D^2 &= (530.59 \text{ GeV})^2 \begin{pmatrix} 1.0 & 5.046 \cdot 10^{-3} e^{-0.01i} & 4.826 \cdot 10^{-3} e^{0.01i} \\ 5.046 \cdot 10^{-3} e^{0.01i} & 1.0 & 5.289 \cdot 10^{-3} e^{0.02i} \\ 4.826 \cdot 10^{-3} e^{-0.01i} & 5.289 \cdot 10^{-3} e^{-0.02i} & 1.0 \end{pmatrix}
 \end{aligned} \tag{22}$$

whose average values show good agreement with (12) and (17). The off-diagonal entries of each squark soft mass-squared are of similar size due to the democratic structure of the Yukawa couplings. The flavor-mixing entries  $m_{\tilde{U}}^2$  are the largest among all three mass squareds.

The flavor-violating entries of Yukawas, trilinear couplings and soft mass-squareds collectively generate radiative contributions  $\gamma^{\mu,d}$ ,  $\Gamma^{\mu,d}$  to the Yukawa couplings below  $M_{weak}$  [11,12]. In fact,  $V_{CKM}^{tree}$  (obtained from  $\mathbf{Y}_{u,d}(M_{weak})$ ) and  $V_{CKM}^{corr}$  (obtained from  $\mathbf{Y}_{u,d}^{eff}$ ) confront as follows:

$$\left\| \left\| V_{CKM}^{tree} \right\| \left\| V_{CKM}^{corr} \right\| \right\| = \left( \begin{array}{cc|cc|cc} \hline 0.9748 & 0.9685 & 0.2229 & 0.2490 & 0.0083 & 0.0085 \\ \hline 0.2229 & 0.2489 & 0.9739 & 0.9674 & 0.0421 & 0.0463 \\ \hline 0.0092 & 0.0104 & 0.0419 & 0.0459 & 0.99908 & 0.99889 \\ \hline \end{array} \right) \quad (23)$$

where left (right) window of  $\square\square$  in  $(i, j)$ -th entry refers to  $|V_{CKM}^{tree}(i, j)|$  ( $|V_{CKM}^{corr}(i, j)|$ ). Obviously,  $|V_{CKM}^{tree}|$  agrees very well with  $|V_{CKM}^{exp}|$  in (7) entry by entry. This qualifies (20) to be the correct high-scale texture given present experimental determination of  $V_{CKM}$  at  $Q = M_Z$ . The most striking aspect of (23) is the fact that supersymmetric threshold corrections push  $V_{CKM}^{tree}$  beyond the experimental bounds. More precisely,  $|V_{CKM}^{corr}(1, 1)|$ ,  $|V_{CKM}^{corr}(1, 2)|$ ,  $|V_{CKM}^{corr}(2, 1)|$ ,  $|V_{CKM}^{corr}(2, 2)|$ ,  $|V_{CKM}^{corr}(3, 3)|$  turn out to have  $17.22\sigma$ ,  $14.21\sigma$ ,  $14.21\sigma$ ,  $15.22\sigma$ ,  $16.40\sigma$  significance levels, respectively. These are obviously far beyond the existing experimental  $1.64\sigma$  significance intervals depicted in (7). As a result, supersymmetric threshold corrections are found to entirely disqualify the high-scale texture (20) to be the correct texture to reproduce the FCNC measurements at the weak scale.

Here, it is worthy of noting that deviation of  $|V_{CKM}^{corr}(i, j)|$  from  $|V_{CKM}^{tree}(i, j)|$  (for  $i, j = 1, 2$ ) turns out to be similar in size for CKM-ruled (see eq. 13) and democratic (see eq. 23) textures. It is smallest for the hierarchical texture (see eq. 18). Therefore, CKM-ruled texture in (10) and democratic one in (20) exhibit a pronounced sensitivity to supersymmetric threshold corrections in comparison to hierarchical texture in (15).

The physical quark fields, which arise after the unitary rotations (5), acquire the masses

$$\overline{\mathbf{M}}_{\mathbf{u}}(M_{weak}) = \text{diag.}(0.055, 1.27, 144.78), \quad \overline{\mathbf{M}}_{\mathbf{d}}(M_{weak}) = \text{diag.}(0.099, 0.27, 2.4) \quad (24)$$

all measured in GeV. In this physical basis for quark fields,  $V_{CKM}^{corr}$  is responsible for charged current interactions in the effective theory below  $M_{weak}$ . The morale of the analysis above is that, the high-scale flavor structures (20) are to be modified in such a way that  $V_{CKM}^{corr}$  agrees with  $V_{CKM}^{exp}$  with sufficient precision. Aftermath, the question is to predict quark masses appropriately at  $Q = M_{weak}$  so that, depending on the particle spectrum of the effective theory beneath, existing experimental values of quark masses at  $Q = M_Z$  are reproduced correctly.

### 3.2 Inclusion of flavor violation from squark soft masses

In this section we extend GUT-scale flavor structures analyzed in Sec. 3.1 by switching on flavor mixings in certain squark soft mass-squareds. In other words, we maintain Yukawa textures to be one of (10), (15) or (20), and examine what happens to CKM prediction if squared masses of squarks possess non-trivial flavor mixings at the GUT scale.

The effective Yukawa couplings  $Y_{u,d}^{eff}$  beneath  $Q = M_{weak}$  receive contributions from all entries of  $\mathbf{m}_{Q,U,D}^2(M_{weak})$  via respective mass insertions [11,12]. Generically, larger the mass insertions larger the flavor violation potential of  $Y_{u,d}^{eff}$ . Consequently, main problem is to determine the relative strengths of on-diagonal and off-diagonal entries of  $\mathbf{m}_{Q,U,D}^2(M_{weak})$  given that they start with a certain pattern of flavor mixings. Take, for instance,  $\mathbf{m}_Q^2$  which evolves with energy scale via (3) at single loop level. Its analytic solution is difficult, if not impossible, given coupled nature of RGEs and further complications brought about by the presence of flavor mixings. However, for an approximate yet instructive analysis, one can consider solving (3) for an infinitesimally small scale change from  $M_{GUT}$  down to  $M_{GUT} - \Delta Q$ :

$$\begin{aligned} \mathbf{m}_Q^2(\Delta t) &= \mathbf{m}_Q^2(0) - \frac{61}{99} \Delta t m_{1/2}^2 \mathbf{1} \\ &+ \Delta t \left\{ \mathbf{m}_Q^2(0) + 2m_0^2 \mathbf{1}, Y_u^\dagger Y_u + Y_d^\dagger Y_d \right\} + 2\Delta t \left( Y_u^{A\dagger} Y_u^A + Y_d^{A\dagger} Y_d^A \right) \end{aligned} \quad (25)$$

with the scale variable  $\Delta t = (4\pi)^{-2} \log(1 - \Delta Q/M_{GUT})$ . Here right-handed squark mass-squareds are taken strictly flavor-diagonal, for simplicity. This approximate solution gives enough clue that diagonal entries of  $\mathbf{m}_Q^2$  tend to take hierarchically large values at the IR due to the gluino mass contribution, mainly. However, its off-diagonal entries do not have such an enhancement source:

$$\begin{aligned}
 (\mathbf{m}_Q^2)_{ij}(\Delta t) &= (\mathbf{m}_Q^2)_{ij}(0) + \Delta t \left[ (\mathbf{m}_Q^2)_{ii}(0) + (\mathbf{m}_Q^2)_{jj}(0) + 4m_0^2 \right] (\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d)_{ij} \\
 &+ \Delta t \left[ (\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d)_{ii} + (\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d)_{jj} \right] (\mathbf{m}_Q^2)_{ij}(0) \\
 &+ \Delta t (\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d)_{6-(i+j)j} (\mathbf{m}_Q^2)_{i6-(i+j)}(0) \\
 &+ \Delta t (\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d)_{i6-(i+j)} (\mathbf{m}_Q^2)_{6-(i+j)j}(0) \\
 &+ 2\Delta t (\mathbf{Y}_u^{\text{A}\dagger} \mathbf{Y}_u^{\text{A}} + \mathbf{Y}_d^{\text{A}\dagger} \mathbf{Y}_d^{\text{A}})_{ij}
 \end{aligned} \tag{26}$$

unless Yukawas or trilinear couplings are given appropriate boundary configurations at the GUT scale. That this is the case can be seen explicitly by considering, for instance, democratic texture for Yukawas (20) together with (9) and strict universality and flavor-diagonality of the soft masses, except

$$\mathbf{m}_Q^2(0) = m_0^2 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{27}$$

which contributes maximally to each term of (26). Even with such a democratic pattern for Yukawas, trilinear couplings and  $\mathbf{m}_Q^2(0)$ , however, one obtains at  $M_{weak} = 1 \text{ TeV}$

$$\mathbf{m}_Q^2 = (533.37 \text{ GeV})^2 \begin{pmatrix} 1.0 & -0.0512 & -0.0510 \\ -0.0512 & 1.0 & -0.0513 \\ -0.0510 & -0.0513 & 1.0 \end{pmatrix} \tag{28}$$

with similar structures for  $\mathbf{m}_U^2$  and  $\mathbf{m}_D^2$ . Alternatively, if one adopts (10) or (15) setups the off-diagonal entries of squark soft mass-squareds at  $M_{weak}$  are found to remain around  $m_0^2$  which are much smaller than the on-diagonal ones. Therefore, Yukawa textures (and hence those of the trilinear couplings) studied in Sec. 3.1 lead one generically to hierarchic textures for squark soft mass-squareds at  $Q = M_{weak}$  irrespective of how large the flavor mixings in  $\mathbf{m}_{Q,U,D}^2(0)$  might be. In fact, predictions for CKM matrix remain rather close to those in Sec. 3.1 above. This is actually clear from (26) where off-diagonal entries of  $\mathbf{m}_{Q,U,D}^2$  are seen to evolve into new mixing patterns via themselves and those of Yukawas and trilinear couplings. In conclusion, evolution of squark soft masses is fundamentally Yukawa-ruled and when Yukawas at the GUT scale are taken to

shoot the measured value of CKM matrix, the mass insertions associated with  $\mathbf{m}_{Q,U,D}^2 (M_{weak})$  are too small to give any significant contribution to  $\mathbf{Y}_{u,d}^{eff}$ .

As follows from (26), for generating sizeable off-diagonal entries for  $\mathbf{m}_{Q,U,D}^2 (M_{weak})$  it is necessary to abandon either Yukawa textures analyzed in Sec. 3.1. or proportionality of trilinear couplings with Yukawas. Therefore, we take Yukawa couplings at the GUT scale precisely as (20), we maintain (9) for both  $\mathbf{Y}_d^A$  and  $\mathbf{Y}_e^A$ , and we take  $\mathbf{m}_U^2 (0)$  and  $\mathbf{m}_D^2 (0)$  strictly flavor-diagonal as in all three case studies carried out in Sec. 3.1. However, we take  $\mathbf{m}_Q^2 (0)$  as (27) above, and  $\mathbf{Y}_u^A$  as

$$\mathbf{Y}_u^A(0) = -150 \text{ GeV} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{29}$$

which certainly violates (9) that enforces trilinears to be proportional to the corresponding Yukawas. Then two-loop RG running from  $Q = M_{GUT}$  down to  $Q = M_{weak}$  gives

$$\begin{aligned} \mathbf{Y}_u^A &= \begin{pmatrix} -262.087 & -259.342 & -260.709 \\ -259.474 & -261.954 & -260.709 \\ -260.688 & -260.674 & -260.735 \end{pmatrix} \\ \mathbf{Y}_d^A &= \begin{pmatrix} -41.435 & 37.091 e^{-0.0127i} & -40.408 e^{0.0124i} \\ -36.171 e^{0.0102i} & 52.230 e^{-0.0019i} & -41.558 e^{0.0228i} \\ 39.983 e^{-0.0300i} & 42.075 e^{-0.0224i} & -41.683 e^{0.0025i} \end{pmatrix} \end{aligned} \tag{30}$$

both measured in GeV at  $M_{weak} = 1 \text{ TeV}$ . Though not shown explicitly, each entry of  $\mathbf{Y}_u^A$  is complex with a phase around  $10^{-7} - 10^{-6}$  in size. On the other hand, squark soft mass-squared at  $Q = M_{weak}$  are given by

$$\begin{aligned}
 \mathbf{m}_Q^2 &= (516.58 \text{ GeV})^2 \begin{pmatrix} 1.0 & -0.13 & -0.13 \\ -0.13 & 1.0 & -0.13 \\ -0.13 & -0.13 & 1.0 \end{pmatrix} \\
 \mathbf{m}_U^2 &= (455.49 \text{ GeV})^2 \begin{pmatrix} 1.0 & -0.3852 & -0.3853 \\ -0.3852 & 1.0 & -0.3853 \\ -0.3853 & -0.3853 & 1.0 \end{pmatrix} \\
 \mathbf{m}_D^2 &= (532.91 \text{ GeV})^2 \begin{pmatrix} 1.0 & 4.34 \cdot 10^{-3} e^{-0.01i} & -4.15 \cdot 10^{-3} e^{0.01i} \\ 4.34 \cdot 10^{-3} e^{-0.01i} & 1.0 & -4.55 \cdot 10^{-3} e^{0.02i} \\ -4.15 \cdot 10^{-3} e^{0.01i} & -4.55 \cdot 10^{-3} e^{0.02i} & 1.0 \end{pmatrix}
 \end{aligned} \tag{31}$$

where small phases in off-diagonal entries of  $\mathbf{m}_Q^2$  and  $\mathbf{m}_U^2$  are neglected. A comparison with (22) reveals spectacular enhancements in mass insertions pertaining  $\mathbf{m}_Q^2$  and  $\mathbf{m}_U^2$ .

The trilinear couplings (30) and squark mass-squareds (31) give rise to non-trivial changes in flavor structures of  $\mathbf{Y}_{u,d}(M_{weak})$  by generating effective Yukawas  $\mathbf{Y}_{u,d}^{eff}$  beneath  $Q = M_{weak}$ . Then the CKM matrix  $V_{CKM}^{tree}$  obtained from  $\mathbf{Y}_{u,d}(M_{weak})$  and  $V_{CKM}^{corr}$  obtained from  $\mathbf{Y}_{u,d}^{eff}$  compare as:

$$\left\| V_{CKM}^{tree} \right\| \left\| V_{CKM}^{corr} \right\| = \begin{pmatrix} \boxed{0.9748} \boxed{0.9637} & \boxed{0.2229} \boxed{0.2668} & \boxed{0.0083} \boxed{0.0080} \\ \boxed{0.2229} \boxed{0.2666} & \boxed{0.9739} \boxed{0.9626} & \boxed{0.0421} \boxed{0.0480} \\ \boxed{0.0092} \boxed{0.0132} & \boxed{0.0419} \boxed{0.0468} & \boxed{0.99908} \boxed{0.99888} \end{pmatrix} \tag{32}$$

where left (right) window of  $\square\square$  in  $(i, j)$ -th entry refers to  $|V_{CKM}^{tree}(i, j)|$  ( $|V_{CKM}^{corr}(i, j)|$ ). Obviously,  $|V_{CKM}^{tree}|$  agrees very well with  $|V_{CKM}^{exp}|$  as was the case in (23). This qualifies (20) to be the correct high-scale texture given present experimental determination of  $V_{CKM}$  at  $Q = M_Z$ . However, implementation of supersymmetric threshold corrections is seen to leave a big impact on certain entries of the physical CKM matrix. Indeed,  $|V_{CKM}^{corr}(1, 1)|$ ,  $|V_{CKM}^{corr}(1, 2)|$ ,  $|V_{CKM}^{corr}(2, 1)|$ ,  $|V_{CKM}^{corr}(2, 2)|$ ,  $|V_{CKM}^{corr}(3, 3)|$  turn out to have  $6.06\sigma$ ,  $23.99\sigma$ ,  $23.89\sigma$ ,  $26.52\sigma$ ,  $4.35\sigma$  significance levels, respectively. These are to be contrasted with standard deviations computed for (23) in Sec. 3.1.3 above. Needless to say, these deviations are far beyond the experimental sensitivities and thus supersymmetric threshold corrections completely disqualify the flavor textures (20) in a way different than (23) due to new structures (27) and (29).

Finally, physical quark fields, which arise after the unitary rotations (5), acquire the masses

$$\overline{\mathbf{M}}_{\mathbf{u}}(M_{weak}) = \text{diag.}(0.138, 1.26, 143.3), \quad \overline{\mathbf{M}}_{\mathbf{d}}(M_{weak}) = \text{diag.}(0.140, 0.304, 2.42) \quad (33)$$

all measured in GeV. These mass predictions are close to those obtained within democratic texture. As in all cases discussed in Sec. 3.1. especially light quark masses fall outside the existing experimental bounds, and choice of the correct high-scale texture must reproduce both  $V_{CKM}^{corr}$  and quark masses in sufficient agreement with experiment.

### 3.3 A purely soft CKM ?

In Sec. 3.1 and 3.2 we have discussed how prediction for the CKM matrix depends crucially on the inclusion of the supersymmetric threshold corrections. This we did by negation *i.e.* we have taken certain Yukawa textures which are known to generate CKM matrix correctly at tree level, and then included threshold corrections to demonstrate how those the would-be viable flavor structures get disqualified.

In this section we will do the opposite *i.e.* we will take a Yukawa texture which is known not to work at all, and incorporate supersymmetric threshold corrections to show how it can become a viable one, at least approximately. For sure, a highly interesting limit would be to start with exactly diagonal Yukawas at the GUT scale and generate CKM matrix beneath  $M_{weak}$  via purely soft flavor violation *i.e.* flavor violation from sfermion soft mass-squareds and trilinear couplings, alone. However, this limit seems difficult to realize, at least for SPS1a' parameter values, since it may require tuning of various parameters, in particular, soft mass-squareds of Higgs and quark sectors [11,12]. Even if this is done by a fine-grained scan of the parameter space, it will possibly cost a great deal of fine-tuning. Indeed, threshold corrections depend on ratios of the soft masses [11], and generating a specific entry of the CKM matrix can require a judiciously arranged hierarchy among various soft mass parameters – a parameter region certainly away from the SPS1a' point.

Therefore, we relax the constraint of strict diagonality and consider instead GUT-scale Yukawa matrices with five texture zeroes which are known to be completely unphysical as they cannot induce the CKM matrix [42]. In fact, this kind of textures has recently been found to arise from heterotic string [43] when the low-energy theory is constrained to be minimal supersymmetric model [44,45]. Consequently, we take Yukawas at  $Q = M_{GUT}$  to be

$$\begin{aligned}
 \mathbf{Y}_u &= \begin{pmatrix} 0 & 9.249 \cdot 10^{-5} & 1.428 \cdot 10^{-3} \\ 1.307 \cdot 10^{-3} & 0 & 0 \\ 0.4675 & 0 & 0 \end{pmatrix} \\
 \mathbf{Y}_d &= \begin{pmatrix} 0 & 9.0 \cdot 10^{-5} & 1.3 \cdot 10^{-3} \\ 1.42 \cdot 10^{-3} & 0 & 0 \\ 0.047 & 0 & 0 \end{pmatrix}
 \end{aligned} \tag{34}$$

with no flavor violation in the lepton sector:  $\mathbf{Y}_e = \text{diag.} (1.9 \cdot 10^{-5}, 0.004, 0.071)$ . Both  $\mathbf{Y}_u$  and  $\mathbf{Y}_d$  are endowed with five texture zeroes, and they precisely conform to the structures found in effective theories coming from the heterotic string [43].

Besides, though left unspecified in [43], we take sfermion mass-squareds strictly flavor-diagonal as in Sec. 3.1, and let  $\mathbf{Y}_e^A$  obey (9). For trilinear couplings pertaining to squark sector we take

$$\begin{aligned}
 \mathbf{Y}_u^A(0) &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -30.469 & -74.029 \\ 0 & -74.029 & -97.406 \end{pmatrix} \\
 \mathbf{Y}_d^A(0) &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -25.241 & -68.185 \\ 0 & -67.545 & -63.990 \end{pmatrix}
 \end{aligned} \tag{35}$$

both measured in GeV. These trilinear couplings do not obey (9); they are given completely independent flavor structures, in particular, they exhibit  $\mathcal{O}(1)$  mixing between second and third generations. The first generation of squarks is decoupled from the rest completely.

Two-loop RG running down to  $Q = M_{weak}$  modifies GUT-scale textures (35) to give

$$\begin{aligned}
 \mathbf{Y}_u^A &= \begin{pmatrix} 0 & -0.157 & -2.426 \\ -1.326 & -75.382 & -183.335 \\ -474.410 & -126.247 & -167.265 \end{pmatrix} \\
 \mathbf{Y}_d^A &= \begin{pmatrix} 0 & -0.231 & -3.341 \\ -3.114 & -78.521 & -212.328 \\ -103.062 & -205.742 & -193.530 \end{pmatrix}
 \end{aligned} \tag{36}$$

both measured in GeV. The texture zeroes in (35) are seen to elevated to small yet nonzero values via RG running. The squark soft mass-squareds, on the other hand, exhibit the following flavor structures at  $M_{weak} = 1 \text{ TeV}$ :

$$\begin{aligned}
 \mathbf{m}_Q^2 &= (560.63 \text{ GeV})^2 \begin{pmatrix} 0.936 & -0.029 & -0.036 \\ -0.029 & 1.051 & -0.049 \\ -0.036 & -0.049 & 1.012 \end{pmatrix} \\
 \mathbf{m}_U^2 &= (523.88 \text{ GeV})^2 \begin{pmatrix} 1.155 & -3.1 \cdot 10^{-4} & -2.9 \cdot 10^{-4} \\ -3.1 \cdot 10^{-4} & 1.107 & -5.5 \cdot 10^{-2} \\ -2.9 \cdot 10^{-4} & -5.5 \cdot 10^{-2} & 0.738 \end{pmatrix} \\
 \mathbf{m}_D^2 &= (548.52 \text{ GeV})^2 \begin{pmatrix} 1.043 & -3.72 \cdot 10^{-4} & -3.54 \cdot 10^{-4} \\ -3.72 \cdot 10^{-4} & 0.997 & -5.322 \cdot 10^{-2} \\ -3.54 \cdot 10^{-4} & -5.322 \cdot 10^{-2} & 0.960 \end{pmatrix}
 \end{aligned} \tag{37}$$

where off-diagonal entries are seen to be hierarchically small so that contributions to  $\mathbf{Y}_{u,d}^{eff}$  from squark soft mass-squareds are expected to be rather small.

The use of Yukawas, trilinear couplings and squark mass-squareds, all rescaled to  $M_{weak} = 1 \text{ TeV}$  via RG running, give rise to modifications in Yukawa couplings after squarks being integrated out. In fact, the CKM matrix  $V_{CKM}^{tree}$  obtained from  $\mathbf{Y}_{u,d}(M_{weak})$  and  $V_{CKM}^{corr}$  obtained from  $\mathbf{Y}_{u,d}^{eff}$  compare as:

$$\left\| V_{CKM}^{tree} \right\| \left\| V_{CKM}^{corr} \right\| = \begin{pmatrix} \boxed{0.9999} \boxed{0.9751} & \boxed{0.0044} \boxed{0.2216} & \boxed{0.0} \boxed{0.0079} \\ \boxed{0.0044} \boxed{0.2218} & \boxed{0.9999} \boxed{0.9742} & \boxed{0.0} \boxed{0.0412} \\ \boxed{0.0} \boxed{0.0014} & \boxed{0.0} \boxed{0.0419} & \boxed{1.0} \boxed{0.99912} \end{pmatrix} \tag{38}$$

where left (right) window of  $\boxed{\quad}$  in  $(i, j)$ -th entry refers to  $|V_{CKM}^{tree}(i, j)|$  ( $|V_{CKM}^{corr}(i, j)|$ ).

It is clear that  $V_{CKM}^{tree}$  by no means qualifies to be a realistic CKM matrix:  $|V_{CKM}^{tree}(i, j)| = 0$  for  $(i, j) = (1, 3), (3, 1), (2, 3), (3, 2)$ ; moreover, Cabibbo angle is predicted to be one order of magnitude smaller. In addition, its diagonal elements turn out to be well outside the experimental limits. However, once supersymmetric threshold corrections are included certain entries are found to attain their experimentally preferred ranges. Indeed,  $|V_{CKM}^{tree}(1, 1)|$  and  $|V_{CKM}^{tree}(3, 1)|$  fall right at their upper bounds, and  $|V_{CKM}^{tree}(1, 3)|$  far exceeds the experimental bound. The predictions for these entries are not good enough; they need to be correctly predicted by further arrangements of the GUT-scale textures. Nevertheless, for the main purpose of illustrating how threshold corrections influence flavor structures at the IR end, the results above are good enough for what has to be shown since all other entries turn out to be in rather good agreement with experimental bounds. The case study illustrated here shows that, even unphysical Yukawa textures with five tex-

ture zeroes, can lead to acceptable CKM matrix predictions once supersymmetric threshold corrections are incorporated into Yukawa couplings.

The corrected Yukawa couplings lead to the following quark mass spectrum:

$$\overline{M}_u(M_{weak}) = \text{diag.}(0.168, 0.93, 151.6), \quad \overline{M}_d(M_{weak}) = \text{diag.}(0.0325, 0.0711, 2.31) \quad (39)$$

all measured in GeV. These predictions are not violatively outside the experimental limits, except for the up quark mass. A rehabilitated choice for the GUT-scale textures (34) should lead to a fully consistent prediction for CKM matrix (with much better precision than in, especially the (1, 3), (3, 1) entries of (38) above) together with precise predictions for quark masses (modulo sizeable QCD corrections while running from  $Q = M_{weak}$  down to hadronic scale).

#### 4 Conclusion

In this work we have studied impact of integrating superpartners out of the spectrum on Yukawa couplings beneath the supersymmetry breaking scale. In Sec. 2 we have outlined the formalism. In Sec. 3.1 we have illustrated effects of threshold corrections with respect to certain high-scale Yukawa textures which are known, at tree level, to lead to experimentally acceptable CKM predictions at TeV scale. In Sec. 3.2 we have switched on flavor violation in squark soft mass-squareds to determine their effects on CKM prediction, and pointed out how important the flavor structure of the trilinear couplings at the GUT scale for squark soft masses to develop sizeable off-diagonal entries. In Sec. 3.3 above our approach was opposite to those in Sec. 3.1 and 3.2 in that we have investigated how threshold corrections can rehabilitate an unacceptable GUT-scale texture by using Yukawa couplings with five texture zeroes. Our main conclusion is that supersymmetric threshold corrections leave observable impact on Yukawa couplings of quarks; confrontation of high-scale textures with experiments at  $Q = M_Z$  should take into account such corrections.

We have focussed mainly on SPS1a' point so as to standardize our predictions for various low-scale couplings and masses. Though not shown explicitly, predictions for Higgs boson masses as well as various other sparticle masses and couplings turn out to be in good agreement with experimental bounds (as a characteristic of SPS1a' point they should agree with laboratory and astrophysical bounds modulo small variations in certain parameters stemming from presence of the flavor violation sources). For supersymmetric parameter regions outside SPS1a', certain parameters are expected to vary significantly, especially at large  $\tan \beta$ . However, scanning of such parameter regions will result mainly in strengthening of the statements arrived at rather conservative SPS1a' point in that supersymmetric threshold corrections should be taken into account in both bottom-up and top-down approaches to supersymmetric flavor problem.

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